

Enhancing the tensor-to-scalar ratio in simple inflation

V. Mukhanov, A. Vikman

ASC, Physics Department LMU, Theresienstr. 37, Munich, Germany

Abstract

We show that in theories with a nontrivial kinetic term the contribution of the gravitational waves to the CMB fluctuations can be substantially larger than that is naively expected in simple inflationary models. This increase of the tensor-to-scalar perturbation ratio leads to a larger B-component of the CMB polarization, thus making the prospects for future detection much more promising. The other important consequence of the considered model is a higher energy scale of inflation and hence higher reheating temperature compared to a simple inflation.

PACS numbers:

I. INTRODUCTION

The main consequence of inflation is the generation of primordial cosmological perturbations [1] and the production of long wavelength gravitational waves (tensor perturbations) [2]. The predicted slightly red-tilted spectrum of the scalar perturbations is at present in excellent agreement with the measurements of the CMB fluctuations [3]. The detection of a small deviation of the spectrum from flat together with the observation of primordial gravitational waves would make us completely confident in early-time cosmic acceleration. The detection of primordial gravitational waves is not easy, but they can be seen indirectly in the B-mode of the CMB polarization (see, for example, [4]). In standard slow-roll inflationary scenarios [5] the amplitude of the tensor perturbations can, in principle, be large enough to be discovered. However, it is only on the border of detectability in future experiments.

There is no problem to modify the inflationary scenarios in a way to suppress the tensor component produced during inflation. In particular, in models such as new inflation [6] and hybrid inflation [7], tensor perturbations are typically small [4]. Moreover, in the curvaton scenario [8] and k-inflation [9], they can be suppressed completely.

An interesting question is whether the gravitational waves can be significantly enhanced compared to our naive expectations. Recently it was argued that the contribution of tensor perturbations to the CMB anisotropy can be much greater than expected [10, 11]. However, it was found in [12] that in the models considered in [10, 11] one cannot avoid the production of too large scalar perturbations and therefore they are in contradiction with observations. It is also possible to produce blue-tilted spectrum of gravitational waves in a so-called superinflation [13], which is, however, plagued by graceful exit problem.

At present there exists no inflationary model with graceful exit where the B-mode of polarization would exceed that predicted by simple chaotic inflation. The purpose of our paper is to show that such models can be easily constructed even within the class of simple slow-roll inflationary scenarios if we allow a nontrivial dependence of the Lagrangian on the kinetic term. These models resemble k-inflation with only difference that here inflation is due to the potential term in the Lagrangian.

II. BASIC EQUATIONS

The generic action describing a scalar field interacting with the gravitational field is

$$S = S_g + S_\varphi = \int d^4x \sqrt{-g} \left[-\frac{R}{16\pi} + p(\phi, X) \right], \quad (1)$$

where R is the Ricci scalar and $p(\phi, X)$ is a function of the scalar field ϕ and its first derivatives

$$X = \frac{1}{2} \nabla_\mu \phi \nabla^\mu \phi. \quad (2)$$

We use Planck units, where $G = \hbar = c = 1$. In the case of the usual scalar field the X -dependence of p is trivial, namely, $p = X - V(\phi)$, while k-inflation and k-essence [4, 9, 14] are based on the non-trivial dependence of p on X . For $X > 0$, variation of the action (1) with respect to the metric gives the energy momentum tensor for the scalar field in the form of an “ideal hydrodynamical fluid”:

$$T_\nu^\mu = (\varepsilon + p) u^\mu u_\nu - p \delta_\nu^\mu. \quad (3)$$

Here the Lagrangian $p(\phi, X)$ plays the role of pressure, the energy density is given by

$$\varepsilon = 2X p_{,X} - p, \quad (4)$$

where $p_{,X} = \partial p / \partial X$, and the “four-velocity” is

$$u_\mu = \frac{\nabla_\mu \phi}{\sqrt{2X}}. \quad (5)$$

Let us consider a spatially flat Friedmann universe with small perturbations:

$$ds^2 = (1 + 2\Phi) dt^2 - a^2(t) [(1 - 2\Phi) \delta_{ik} + h_{ik}] dx^i dx^k, \quad (6)$$

where Φ is the gravitational potential characterizing scalar metric perturbations and h_{ik} is a traceless, transverse perturbations describing the gravitational waves. The scale factor $a(t)$ satisfies the Friedmann equation

$$H^2 \equiv \left(\frac{\dot{a}}{a} \right)^2 = \frac{8\pi}{3} \varepsilon, \quad (7)$$

where the dot denotes the derivative with respect to time t . Taking into account that for the homogeneous background scalar field $X = \frac{1}{2} \dot{\phi}^2$ and that the energy conservation law is

$$\dot{\varepsilon} = -3H(\varepsilon + p), \quad (8)$$

we obtain the following equation for the scalar field

$$\ddot{\phi} + 3c_S^2 H \dot{\phi} + \frac{\varepsilon_{,\phi}}{\varepsilon_{,X}} = 0, \quad (9)$$

where the “speed of sound” is

$$c_S^2 \equiv \frac{p_{,X}}{\varepsilon_{,X}} = \left[1 + 2X \frac{p_{,XX}}{p_{,X}} \right]^{-1}. \quad (10)$$

Considering small scalar metric perturbations one can show that c_S is in fact the speed of propagation of the cosmological perturbations [4, 15]. The stability condition with respect to the high frequency cosmological perturbations requires $c_S^2 > 0$.

III. GENERALIZED SLOW-ROLL INFLATION

It follows from (4) that inflation can be realized if the condition $Xp_{,X} \ll p$ is satisfied for a sufficiently long time interval. This can be done in two ways. Considering the canonical scalar field with $p = X - V(\phi)$, one can take a flat potential $V(\phi)$ so that $X \ll V$ for more than 75 e-folds. This is the standard slow-roll inflation [5] and in this case $c_S = 1$. The other possibility is the k-inflation [9], where p is a weakly dependent function of X , so that $p_{,X}$ is small. Here inflation is entirely based on the kinetic term and it can take place even if the field is running very fast (X is large); typically $c_S^2 \ll 1$ for k-inflation.

In this paper we consider a slow-roll inflation with a flat potential but in theories with a nontrivial kinetic term. To best of our knowledge this possibility has been ignored in the literature until now. However such models are very interesting because, as we will see, they allow us to have $c_S^2 > 1$ during inflation and thus enhance the tensor-to-scalar ratio for the perturbations produced.

The Lagrangian $p(\phi, X)$ is manifestly Lorentz-invariant and a superluminal speed of cosmological perturbations does not contradict the principles of relativity. In fact, the superluminal propagation is possible only in the presence of the time-dependent homogeneous scalar field, which determines a preferable coordinate frame. Only in this frame the speed can exceed the speed of light. As a result no violation of causality is possible.

For simplicity let us consider theories with Lagrangian

$$p = K(X) - V(\phi). \quad (11)$$

In this case

$$\varepsilon = 2XK_{,X} - K + V, \quad (12)$$

and the equation for scalar field becomes

$$\ddot{\phi} + 3c_S^2 H \dot{\phi} + \frac{V_{,\phi}}{\varepsilon_{,X}} = 0. \quad (13)$$

It is clear that if the slow-roll conditions

$$XK_{,X} \ll V, \quad K \ll V, \quad \left| \ddot{\phi} \right| \ll \frac{V_{,\phi}}{\varepsilon_{,X}}$$

are satisfied for at least 75 e-folds then we have a successful slow-roll inflation due to the potential V . In contrast to ordinary slow-roll inflation one can arrange here practically any speed of sound c_S^2 by taking an appropriate kinetic term. For example, for

$$K(X) = \alpha X^\beta,$$

we obtain from (10)

$$c_S^2 = 1/(2\beta - 1).$$

and if $\beta \rightarrow 1/2$, then $c_S^2 \rightarrow \infty$. Therefore by considering nontrivial kinetic terms $K(X)$ one can have, in principle, an arbitrarily large c_S , which thus becomes an additional free parameter of the theory. The crucial point is that the amplitude of the final scalar perturbations (during the postinflationary, radiation-dominated epoch) depends on c_S (see, [4]):

$$\delta_\Phi^2 \simeq \frac{64}{81} \left(\frac{\varepsilon}{c_S(1 + p/\varepsilon)} \right)_{c_S k \simeq Ha}, \quad (14)$$

while the ratio of tensor to scalar amplitudes on supercurvature scales is given by

$$\frac{\delta_h^2}{\delta_\Phi^2} \simeq 27 \left(c_S \left(1 + \frac{p}{\varepsilon} \right) \right)_{k \simeq Ha}. \quad (15)$$

Here it is worthwhile reminding that all physical quantities on the right hand side of Eqs. (14) and (15) have to be calculated during inflation at the moment when perturbations with wave number k cross corresponding Horizon: $c_S k \simeq Ha$ for (14) and $k \simeq Ha$ for (15) respectively. The amplitude of the scalar perturbations δ_Φ is a free parameter of the theory which is taken to fit the observations. Therefore, in models where $c_S^2 > 1$, the energy scale of inflation must be higher than in ordinary slow-roll inflation with canonical kinetic term. Moreover, it follows from (15) that the tensor-to-scalar ratio can be arbitrarily enhanced in such models.

IV. SIMPLE MODEL

As a concrete example from the class of theories with enhanced tensor contribution let us consider a simple model with Lagrangian

$$p(\phi, X) = \alpha^2 \left[\sqrt{1 + \frac{2X}{\alpha^2}} - 1 \right] - \frac{1}{2} m^2 \phi^2, \quad (16)$$

where constant α is a free parameter. For $2X \ll \alpha^2$ one recovers the Lagrangian for the usual free scalar field. The function p is a monotonically growing concave function of X ,

$$p_{,X} = \left(1 + \frac{2X}{\alpha^2} \right)^{-1/2} > 0, \quad p_{,XX} = -\frac{1}{\alpha^2} \left(1 + \frac{2X}{\alpha^2} \right)^{-3/2} < 0,$$

and the corresponding energy density,

$$\varepsilon = \alpha^2 \left[1 - \left(1 + \frac{2X}{\alpha^2} \right)^{-1/2} \right] + \frac{1}{2} m^2 \phi^2, \quad (17)$$

is always positive. The effective speed of sound,

$$c_s^2 = \frac{p_{,X}}{\varepsilon_{,X}} = 1 + \frac{2X}{\alpha^2}, \quad (18)$$

is larger than the speed of light, approaching it as $X \rightarrow 0$.

In the slow-roll regime and for p given in (16), equations (7), (9) simplify to

$$H \simeq \sqrt{\frac{4\pi}{3}} m \phi, \quad 3p_{,X} H \dot{\phi} + m^2 \phi \simeq 0. \quad (19)$$

Taking into account that $\dot{\phi} = -\sqrt{2X}$, we infer that during inflation $\dot{\phi}$ is constant and

$$\frac{2X}{\alpha^2} = \left(\frac{12\pi\alpha^2}{m^2} - 1 \right)^{-1}. \quad (20)$$

The speed of sound

$$c_s \simeq c_\star = \left(1 - \frac{m^2}{12\pi\alpha^2} \right)^{-1/2} \quad (21)$$

can be arbitrarily large during inflation if we take $12\pi\alpha^2 \rightarrow m^2$. Note, however, that $12\pi\alpha^2 > m^2$ is necessary for the existence of the slow-roll solution. Hereafter we will use c_\star as a parameter instead of α . One can easily find that during the slow-roll regime

$$\dot{\phi} \simeq -\frac{mc_\star}{\sqrt{12\pi}}, \quad (22)$$

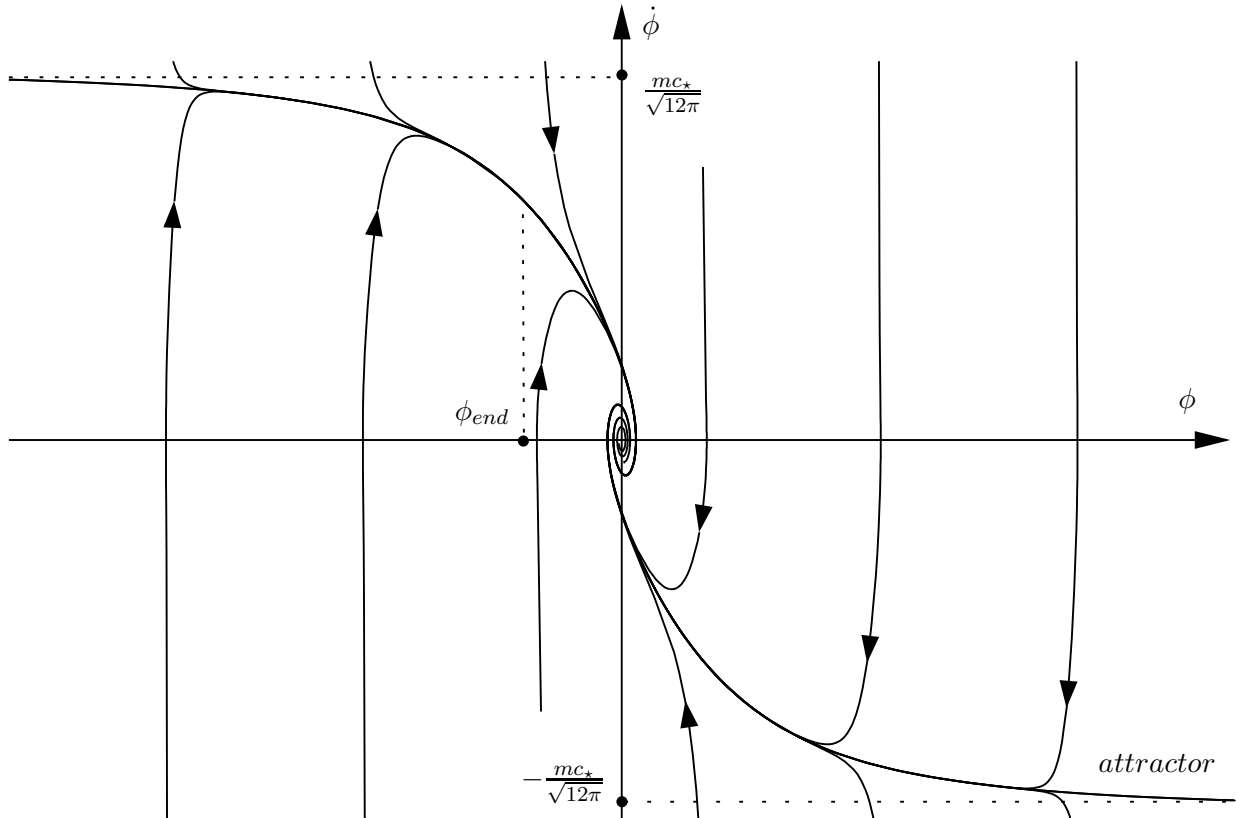


FIG. 1: Numerically obtained phase portrait for the system with the parameters $m = 1.5 \cdot 10^{-7}$, $c_\star = 3.67$. The field value $\phi_{end} = 0.32$ corresponds to the end of acceleration when $w = -1/3$.

and the pressure and energy density are given by

$$p \simeq m^2 \left(\frac{1}{12\pi} \frac{c_\star^2}{1 + c_\star} - \frac{\phi^2}{2} \right), \quad \varepsilon \simeq m^2 \left(\frac{1}{12\pi} \frac{c_\star}{1 + c_\star} + \frac{\phi^2}{2} \right), \quad (23)$$

respectively. The inflation is over when

$$\frac{\varepsilon + p}{\varepsilon} \simeq \frac{c_\star}{6\pi\phi^2} \quad (24)$$

becomes of order unity, that is, at $\phi \sim \sqrt{c_\star/6\pi}$. After that the field ϕ begins to oscillate and decays. The detailed analysis of dynamics and stability of the model will be given in [16]. Here we restrict ourselves to illustration of the above analysis by a phase diagram Fig. 1 obtained numerically.

To determine $a(\phi)$ we use (22) to rewrite the first equation in (19) as

$$-\frac{mc_\star}{\sqrt{12\pi}} \frac{d \ln a}{d\phi} \simeq \sqrt{\frac{4\pi}{3}} m\phi, \quad (25)$$

and obtain

$$a(\phi) \simeq a_f \exp\left(\frac{2\pi}{c_\star}(\phi_f^2 - \phi^2)\right), \quad (26)$$

where a_f and $\phi_f \sim \sqrt{c_\star/6\pi}$ are the values of the scale factor and the scalar field at the end of inflation. Given a number of e-folds before the end of inflation N , we find that at this time

$$\frac{2\pi\phi^2}{c_\star} \simeq N, \quad (27)$$

and, hence,

$$\frac{\varepsilon + p}{\varepsilon} \simeq \frac{1}{3N}$$

does not depend on c_\star . Thus, for a given scale, which crosses the Hubble scale N e-folds before the end of inflation, the tensor-to-scalar ratio is

$$\frac{\delta_h^2}{\delta_\Phi^2} \simeq 27c_\star \left(1 + \frac{p}{\varepsilon}\right) \simeq \frac{9c_\star}{N}. \quad (28)$$

It is clear that by choosing α close to the critical value $m/\sqrt{12\pi}$ we can have a very large c_\star and consequently enhance this ratio almost arbitrarily.

V. CONCLUSIONS

We have shown above that in theories where the Lagrangian is a nontrivial, nonlinear function of the kinetic term, the scale of inflation can be pushed to a very high energies without coming into conflict with observations. As a result, the amount of produced gravitational waves can be much larger than is usually expected. If such a situation were realized in nature then the prospects for the future detection of the B-mode of CMB polarization are greatly improved. Of course, the theories where this happens are somewhat “fine-tuned”. Namely, the corresponding Lagrangian $p(X, \phi)$ must generically satisfy the condition, $p_{,X} + 2Xp_{,XX} \ll p_{,X}$, during inflation. For the particular model (16) this means that $12\pi\alpha^2 - m^2 \ll 12\pi\alpha^2$. This fine-tuning of the parameters of the theory should not be confused, however, with the fine-tuning of the initial conditions. In fact, the true theory of nature is unique and, for example, Lagrangian (16), where $12\pi\alpha^2$ is not very different from m^2 looks even more attractive because it has fewer free parameters. Therefore, future observations of the CMB fluctuations are extremely important since they will restrict the number of possible candidates for the inflaton. The other peculiar properties of the generalized slow-roll inflation will be considered in [16].

Acknowledgements

We are grateful to Matthew Parry for very useful discussions and Marco Baldi, Fabio Finelli, Sabino Matarrese for useful correspondence.

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